LESSON 4.3b

Polynomial Synthetic Divison

Today you will:

- Divide polynomials using synthetic division
- Practice using English to describe math processes and equations

Core Vocabulary:

- synthetic polynomial division, p. 175

$$\boldsymbol{dividend \div divisor = quotient + \frac{remainder}{divisor}}$$

Previous:

- dividend - the number being divided … the number on top

$$\boldsymbol{\begin{array}{r|l} \multicolumn{1}{r}{} & quotient \\ \hline divisor & dividend \end{array}}$$

- divisor - the “divide by” number … the number on the bottom
- quotient - the result of the division, the answer

$$\boldsymbol{\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor}}$$

- remainder - what is left over when the divisor does not evenly go into the dividend

Synthetic Division

A shortcut for dividing polynomials by binomials of the form $(x - k)$.

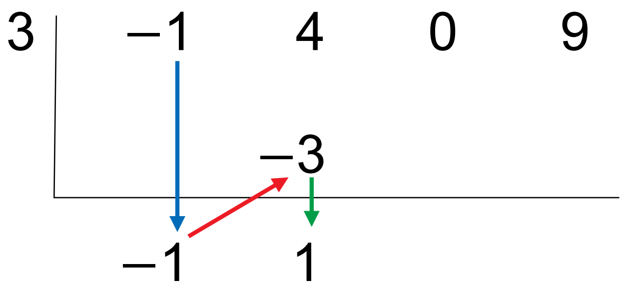
Divide $-x^3 + 4x^2 + 9$ by $x - 3$.

SOLUTION

Step 1 Write the coefficients of the dividend in order of descending exponents. Include a “0” for the missing x-term. Because the divisor is $x - 3$, use $k = 3$. Write the k-value to the left of the vertical bar.

$$k\text{-value} \longrightarrow 3 \;\begin{array}{|rrrr} -1 & 4 & 0 & 9 \\ & & & \\ \hline \end{array} \longleftarrow \text{coefficients of } -x^3 + 4x^2 + 9$$

Step 2 Bring down the leading coefficient. Multiply the leading coefficient by the k-value. Write the product under the second coefficient. Add.



$$3 \;\begin{array}{|rrrr} -1 & 4 & 0 & 9 \\ & -3 & & \\ \hline -1 & 1 & & \end{array}$$

Step 3 Multiply the previous sum by the k-value. Write the product under the third coefficient. Add. Repeat this process for the remaining coefficient. The first three numbers in the bottom row are the coefficients of the quotient, and the last number is the remainder.

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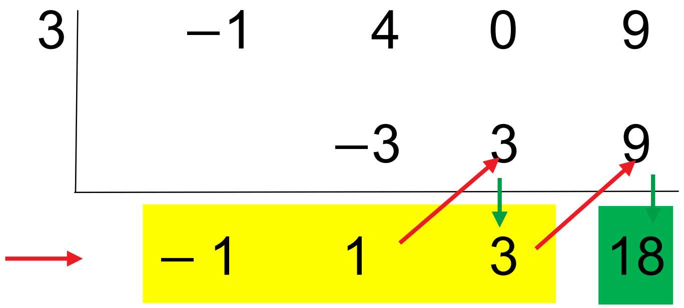

$$\begin{array}{r|rrrr} 3 & -1 & 4 & 0 & 9 \\ & & -3 & 3 & 9 \\ \hline & -1 & 1 & 3 & \boxed{18} \end{array}$$

coefficients of quotient ⟶ $-1 \quad 1 \quad 3$

18 ⟵ remainder

$$\frac{-x^3 + 4x^2 + 9}{x - 3} = -x^2 + x + 3 + \frac{18}{x - 3}$$

Divide $3x^3 - 2x^2 + 2x - 5$ by $x + 1$.

SOLUTION

Use synthetic division. Because the divisor is $x + 1 = x - (-1)$, $k = -1$.

$$\begin{array}{r|rrrr} -1 & 3 & -2 & 2 & -5 \\ & & -3 & 5 & -7 \\ \hline & 3 & -5 & 7 & -12 \end{array}$$

$$\frac{3x^3 - 2x^2 + 2x - 5}{x + 1} = 3x^2 - 5x + 7 - \frac{12}{x + 1}$$

STUDY TIP

Note that dividing polynomials does not always result in a polynomial. This means that the set of polynomials is *not* closed under division.

Divide $x^3 - 3x^2 - 7x + 6$ by $x - 2$.

SOLUTION

Use synthetic division. Because the divisor is $x - 2$, $k = 2$.

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -7 & 6 \\ & & 2 & -2 & -18 \\ \hline & 1 & -1 & -9 & -12 \end{array}$$

$$\frac{x^3 - 3x^2 - 7x + 6}{x - 2} = x^2 - x - 9 - \frac{12}{x - 2}$$

Divide $2x^3 - x - 7$ by $x + 3$.

SOLUTION

Use synthetic division. Because the divisor is $x + 3$, $k = -3$.

$$\begin{array}{r|rrrr} -3 & 2 & 0 & -1 & -7 \\ & & -6 & 18 & -51 \\ \hline & 2 & -6 & 17 & -58 \end{array}$$

$$\frac{2x^3 - x - 7}{x + 3} = 2x^2 - 6x + 17 - \frac{58}{x + 3}$$

Homework

Pg 177, #5-18, 39 (using synthetic division for all problems)